Auction Procedure Guide

Disclaimer:

This document was prepared to aid the analysis of the rules and procedures of the auction, including practical examples wherever warranted,

This document does not from part of the auction regulation (hereinafter the Regulation), which prevails at all times and for all effects.

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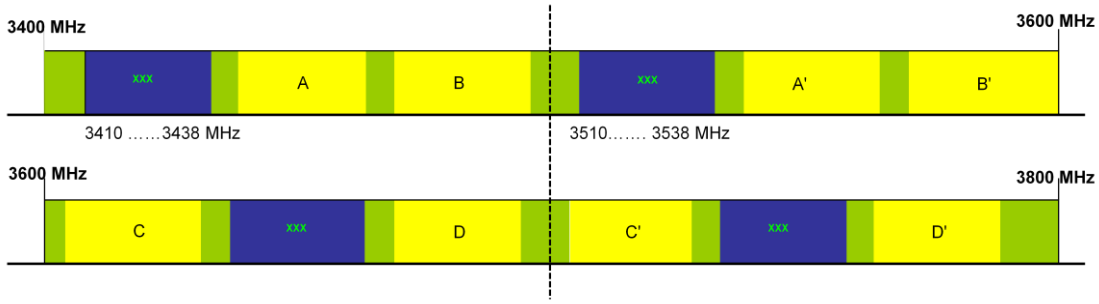
1 Auction Model

The auction is combinatorial, with two stages - Distribution stage and Assignment stage - both of which consist of a single round with sealed bidding and are based on the Second Price Rule.

The auction is combinatorial insofar as it allows participating entities to bid on different lots in different geographical areas. In this auction each bidder may bid on up to 4 lots in each of the 9 geographical areas.



The principle of the second price rule means that bidders do not necessarily pay the maximum price they are willing to pay (i.e., the value of the bid), but an amount which is less than this and which still provides the same winner determination result.

This principle is applied in the two rounds of bidding, thereby determining:

- The base price payable as a result of the distribution phase;
- The top-up price payable as a result of the assignment stage;

The purpose of using the principle of the second-price rule is to encourage the calculation of the real value that each bidder assigns to the spectrum in question and to remove the strategic complexity of bidding. This follows from the exclusion of the cost variable in calculating the optimal bid amount, with the only variable present based on the probability of winning. Therefore, the correct incentive is generated for the revelation of the true value the market assigns to the auctioned spectrum.

The two bidding rounds in the auction will take place in the following stages:

- Distribution stage - set out in articles 16 to 22 of the Regulation;
- Assignment stage - set out in articles 23 to 28 of the Regulation.

2 Applications

It is established in article 12 of the Regulation, that applications must be formalized by written request addressed to the Chairman of the Management Board of ICP-ANACOM, containing identification of the applicant, a reference to these regulations, the date and the applicant's signature.

This article also stipulates that application requests, together with the items identified in article 13, shall be delivered to the Public Attendance Service at the headquarters of ICP-ANACOM, in return for receipt of delivery, on working days between 9 am and 4 pm.

The Committee shall verify the application requests and, based on the requirements set out in articles 8, 12 and 13, shall propose the admission or exclusion of said requests to the Management Board of ICP-ANACOM, notifying all applicants of such, under the terms of article 15.

3 Deposit

Article 9 establishes that, in order to safeguard the commitment assumed with the submission of the applications and the obligations inherent to the auction, the applicants shall provide a deposit by bank guarantee or an insurance deposit in favour of ICP-ANACOM.

The deposit shall cover at least 50% of the sum of the reserve prices of the lots in the relevant bid in the distribution stage, or should there be an exemption of this stage, it shall be 50% of the sum of the reserve prices of the lots in the areas selected by the sole bidder under the terms of article 16. Additionally, a relevant bid is that whose sum of the reserve prices of the lots comprising it is the highest among all the bids which are submitted by the applicant.

In other words, after the applicant defined its strategy in terms of number of lots by geographical area, and formulating/developing options for the bidding it intends to submit, he should choose as relevant that (bid) where the sum of the reserve prices - of the lots it comprises - is the highest. Thus, the amount of the guarantee deposit must be at least 50% of the sum of reserve prices of the lots comprised by the relevant bid.

As an example, candidate A may wish to obtain spectrum in three areas, and defined its bidding strategy as follows:

Bidding Option	Number of lots by area			Sum of reserve prices (thousands of €)
	Area 1	**Area 2**	**Area 3**	
A # 1	3	2	1	1650
A # 2	3	2	0	1500
A # 3	3	1	1	1350
A# 4	3	1	0	1200
A # 5	3	0	0	900

Regardless of the amount that he is willing to pay in each bidding option, the applicant will have to make a deposit corresponding to at least 50% of €1650 000, i.e. € 825 000.

Finally, article 9 defines the conditions under which the deposits may be withdrawn, particularly when the application has not been admitted; subsequent to receipt of the notification referenced in paragraph 3 of article 15; in the case of application

withdrawal according to the terms allowed in clause 4 of article 15, following notification of such intent. In the case of non-winning bidders, ICP-ANACOM shall release the guarantee deposits within 10 working days following the date on which the results of the distribution stage are disclosed, pursuant to article 22.

4 Second price rule

In the traditional Vickrey auction model, the second price rule requires that the winning bidder pays the price of the second highest bid. Since this model is based on a single and indivisible object, the second price can be calculated immediately. The amount payable is the bid amount minus the Vickrey discount, which is the difference between the amount of winning bid and second highest bid amount. However, given that the model proposed by ICP-ANACOM is combinatorial, i.e., the object is not unique, the application of the second price rule is more complex.

The principle underlying the determination of the amount payable by the winning bidder is the same as with the traditional second-price rule, i.e. the winning bidders pay the lowest possible amount that continues to ensure that they retain their winning positions.

Based on this principle, an iterated algorithm will determine the discount that will be applied to each winning bidder, thereby obtaining the respective base prices (in the bidding round of the distribution stage) and the top-up price (in the bidding round of the assignment stage).

Bearing in mind that this type of auction is combinatorial, the iterated algorithm will give the amount associated with each bidder, calculating:

1) The total amount of winning bids: V_t. This is the total aggregated value of the chosen bids of each bidder;

2) The discount value of each winning bidder (Vickrey Discount - DV_i): the alternative amount that ICP-ANACOM would receive if this bidder had not participated (V_{ai}) will be calculated. The difference between the total value of the winning bids (V_t) and the alternative total value of a given winning bidder (V_{ai}) is the so called Vickrey discount for this winning bidder ($DV_i = V_t - V_{ai}$). Note that (V_t) will be the sum of the winning bid in case of a simple auction, (DV_i) the discount to apply to the winning bid and (V_{ai}) would be the second highest amount bid, i.e., the amount payable by the winning bidder;

3) Having obtained the Vickrey Discounts for each winning bidder (DV_i) the discounts corresponding to the combinations of winning bids are calculated. The process for calculating a given combinatorial discount is similar to the process for calculating individual discounts, i.e., obtained as the difference between the total amount of the winning bids (V_t) and the amount corresponding to the sum of the bids that would win if the bidders in question had not participated.

Therefore, in a scenario where 3 winning bidders have been determined as such (A, B and C), the first step is to find the values: V_t and DV_A, DV_B and DV_C. The second step is to find the value of DV_{A+B}; DV_{A+C}, DV_{B+C} and $DV_{A+B+C.}$

If the sum of the individual discounts is equal to or less than the combinatorial discounts, or in this example, if all of the following conditions are simultaneously met:

a) $DV_A + DV_B <= DV_{A+B}$

b) $DV_A + DV_C <= DV_{A+C}$

c) $DV_B + DV_C <= DV_{B+C}$

d) $DV_A + DV_B + DV_C < = DV_{A+B+C}$,

the discount applied to each winning bid is the Vickrey discount DV_i calculated in point 2).

Where the sum of individual discounts exceeds the discount of the respective combination, it will be necessary to obtain the minimization of the squares of the differences between the Vickrey discounts and the discounts actually granted, subject to the restriction that these Vickrey discounts, added together, are equal to or below the combinatorial discounts. In our example, if all the conditions of point 4 are met, except for a), the solution of the problem is found by minimizing the following expression:

$$\text{Min } [(D_A-DV_A)^2 + (D_B-DV_B)^2], \text{ s.a. } DV_A + DV_B < = DV_{A+B}$$

5 Distribution Stage

5.1 Bids in the Distribution Stage

In the Distribution Stage, bidders express their preferences and the values that they assign to each package using the bidding form for the stage in question. It is recalled that, as defined in article 2, a package is a combination of the number of lots that a bidder seeks to be granted in each region.

This stage will only take place in the event that more than one applicant is admitted. Otherwise, the sole applicant is given the opportunity to choose the desired lot, or package of lots, at the amount that corresponds to the sum of reserve prices of each lot making up this package, in light of the value of deposit provided.

In the distribution stage, bidders submit their bids using a specific form. This form, as well as all the procedures to be followed, will be made available by the Committee to all applicants via email in advance. The date, start and end time of the single round of bidding will be sent to eligible bidders and will be published in a notice by ICP-ANACOM in the press and on its website.

For each bid, indication must be given, for each geographical area, of the number of lots forming the package of lots and the amount that the bidder is willing to pay for this package of lots.

It should be noted that the amount of each bid must be equal to or greater than the sum of reserve prices of the lots that make up the package.

It should also be noted that the lots included in the package must have an aggregate reserve price which is less than twice the amount of the deposit provided (as set out in paragraphs 1 and 2 of article 9).

The following figure shows a form filled in by a given bidder A, who, in this example, decided to limit the number of his bids to 5 only. It also shows, for each bid, the amount the bidder is willing to pay for the combination of lots. In this example, the deposit made by the bidder must be no less than 825 thousand euros. In the event that the bidder has made a deposit, for example, of 700 thousand euros, bidding

options 1 and 2 would be rejected under point d) of paragraph 1 of article 19. The reserve prices for each geographical area are listed in article 3 of the Regulation.

Bidding Option	Number of lots			Amount (thousand €)
	Area 1	Area 2	Area 3	
A # 1	3	2	1	1 725
A #2	3	2	0	1 600
A # 3	3	1	1	1 400
A # 4	3	1	0	1 300
A # 5	3	0	0	200

It should be noted that in this example the form includes only 3 of the 9 geographical areas, while the form used in the auction will include all 9 geographical areas, regardless of the areas where bids are made.

- The preferences of bidder A can be summarized as follows:
  - His business strategy will be developed mainly in geographical areas 1 and 2, and possibly in area 3;
  - He wants to obtain 3 lots in area 1, regardless of who wins in the two other areas;
  - In area 2 the bidder wants to secure 1 or 2 lots, and the value he attributes to his strategy and to his bid will depend on the number of lots won in this area.
  - Area 3 is not central to his strategy, however he would value the lot obtained in this area.

Bidding option number 5 will be rejected in its entirety, since the amount is less than the sum of reserve prices (see article 19), i.e., the minimum bid required for option 5 to be valid is 900 thousand euros.

If the field regarding the number of desired lots in a given area is left blank, the number of desired lots is assumed to be zero.

It should be noted that the figures given on the form do not need to be different for all the bids. In fact, the amounts reflect the valuation given by each bidder to a particular package of lots.

Where there are two bidding options that with different amounts for the same package of lots, the one with the highest amount shall prevail, in accordance with paragraph 3 of article 19.

Each bidder has sole responsibility for any errors made in their bids, as established by point f) of article 18.

5.2 Determination of the winners and the respective base prices in the distribution stage

An algorithm determines the winning bidders using the combination of bids which generates the largest total value among all the valid bids submitted by all bidders.

It should be noted that the bids submitted by each bidder are mutually exclusive, i.e., in the process of determining the winner just one and one bid only may be selected from each bidder, pursuant to point b) of paragraph 1 of article 20.

Analysing the bids submitted by each bidder individually, and applying the criteria set out in articles 18 and 19, the bids will be maintained or removed from the form, which will then be submitted for processing.

Given the definition of bidding set out in points j) of article 2 and b) of paragraph 1 of article 20 of the Regulation, the winner determination process means that a single bid may be chosen per bidder from among the bids presented on the form.

The winning combination will be chose according to the criterion of the highest generated value.

Accordingly, the winning bidders are determined by finding all the bids submitted by all bidders, which simultaneously:

- Reconcile the number of lots available in each geographical area with those desired by the bidders;
- Generate the greatest value when the amounts bid are added together.

Finally, only one bid is selected from each bidder.

In order to facilitate the understanding of how winners are determined in the distribution phase and how the base prices are calculated, three simple examples are presented below. Note that the three examples assume away the existence of reserve prices. It was decided not to include these prices for the sake of simplicity, but the steps for determining both winners and base prices would be basically identical. In fact, the lack of reserve prices can be interpreted as being the same as if these were set at null. In accordance with article 21, the base price may not be less than the sum of reserve prices of the lots won, which means there is an implicit restriction in the examples that follow, in that the prices paid by bidders can never be negative.

The examples given below are given for illustrative purposes only, since the numbers of lots and geographical areas do not correspond to the auction in question (4 lots in each of the 9 geographic areas). The aim is to facilitate the understanding of some mechanisms of the auction.

Example 1:

Number of lots: 10

Number of regions: 1

Number of bidders: 5

Bids:

Bidder	Number of lots	Amount bid
A	3	€35
B	3	€25
C	4	€40
D	2	€15
E	4	€35

In order to determine the winning bidders, it is first necessary to identify the bids whose sum of values is highest, taking into account that there is a limit of 10 lots to be allocated.

In this example it is easy to see that the winning combination is the one combining the bids of bidders A, B and C, where the sum of the amounts bid is €100.

In order to calculate the base price of each of the winning bidders A, B and C (price to be paid in case there is no assignment stage), it is necessary to calculate the Vickrey discounts and the combinatorial discounts, as mentioned in the section on the second price rule.

The Vickrey discounts correspond to the maximum discounts which may be applied to the winning bids and which continue to ensure that the respective bidders, alone, remain winners. As such, they represent the opportunity cost of a particular bidder, that is, the cost of a determined number of lots, were these won by the non-winning bidder(s) which offered the highest amount for the same lots - in the absence of such non-winning bidder(s), the opportunity cost is zero. The Vickrey discount for a determined winning bid "i" is obtained by calculating the difference between the sum of the amounts of the winning bids and the total sum of the amounts of the winning bids excluding bidder "i", including if possible one or more of the non-winning bidders, which may however be accommodated with the exit of bidder "i".

Therefore, in this example, if bidder "A" had not participated in the auction, there would be 3 unallocated lots. With B and C remaining winners, and given that bidder E requires 4 lots, only the bid of bidder D can be accommodated. As the amount of D's bid is € 15 and the amount of A's bid is € 35, the difference in the sum of the amounts of the winning bids - with and without A's participation - is € 20: this is the maximum discount that may be applied to A's bid while ensuring that A would remain the winner.

Applying this reasoning to the three winning bidders, the Vickrey discount is obtained. The discounts to be effectively applied to each of the winning bids will be, at most, equal to the Vickrey discounts (and at least equal to zero). Therefore, we have three initial restrictions:

1) $D_a \leq DV_a$
2) $D_b \leq DV_b$
3) $D_c \leq DV_c$

Where:

D_a, D_b and D_c represent, respectively, the discounts to be applied to the bids of A, B and C, and

DV_a, DV_b and DV_c represent, respectively, the Vikrey discounts of A, B and C.

Substituting the values of the Vickrey discounts, we get:

1) $D_a \leq DV_a = €20$
2) $D_b \leq DV_b = €10$
3) $D_c \leq DV_c = €5$

The restrictions presented, while necessary, are not sufficient to determine the final discounts to be applied to every winning bid. To understand why, it is useful to recall points a) and b) of paragraph 1 of article 21 of the Regulation governing the present auction.

Conditions 1), 2) and 3), alluded above, guarantee - in this example where the reserve prices are zero - that the condition set forth in point b) of paragraph 1 of article 21 is fulfilled, i.e. that "the amount of the bid discount for each winning bidder is not greater than the difference between the value of the winning bid and the sum of the reserve prices associated with the lots contained in the winning bid."

However, these restrictions do not guarantee compliance with the condition, set out in point a) of article 21, that is, that "the procedure of winner determination produces the same result ... [considering] all possible subsets of winning bidders". For this example, failure to comply with the latter condition means that it was possible, after the application of discounts, that the subset of winning bids would generate a value less than the total generated by another subset determined as non-winning (before the application of the discounts). This would violate the provisions of paragraph 1 of article 20, namely that "the winning bids are obtained by establishing the combination of bids that have the highest aggregate bid amount from all the valid bids presented by all the bidders(…)".

It is therefore essential to ensure that after the discounts are applied to the individual bids, these bids remain as winners, whether considered in isolation or as subsets. The concept to be followed is the same as that considered when calculating the Vickrey discounts, i.e., the maximum discount applicable to a particular subset of winning bids is the opportunity cost of these bidders not participating in the auction.

Returning to the example, bidder A and bidder B have to pay, together, at least as much as the best alternative. Since bidder A and bidder B won, together, 6 lots, the best alternative would accommodate bidders D and E. As the sum of the latter bids amounts to €50, bidders A and B may never pay less than € 50, and since they bid €60 in total, the maximum discount to be applied to the *set* of the two bids is €10. Following the same reasoning for all sub-sets of winning bids, two by two, we get 3 new restrictions:

4) $D_a + D_b \leq DV_{a,b}$

5) $D_a + D_c \leq DV_{a,c}$

6) $D_b + D_c \leq DV_{b,c}$

Substituting with the respective opportunity costs, we get:

4) $D_a + D_b \leq DV_{a,b} = €10$

5) $D_a + D_c \leq DV_{a,c} = €25$

6) $D_b + D_c \leq DV_{b,c} = €15$

Finally, a seventh restriction needs to be imposed, with reference to the whole set of winning bids. If bidders A, B and C had not participated in the auction, the bids of D and E would win. As such, bidders A, B and C cannot pay *together* less than €50, which is the opportunity cost of the totality of winning bids. Since the amounts of the three winning bids total €100, the sum of the discounts to be applied to each winning bid may not exceed € 50.

7) $D_a + D_b + D_c \leq DV_{a,b,c}$

Substituting with the obtained value results in:

7) $D_a + D_b + D_c \leq DV_{a,b,c} = €50$

There are therefore 7 restrictions which must be satisfied. Recalling that the goal is to obtain the largest possible discounts, provided that all winning bidders remain as such after their implementation, we can start off by testing restrictions 4) to 7) with the highest values possible for individual discounts, i.e. the Vickrey discounts obtained with the calculation of restrictions 1) to 3). Since these values satisfy, by definition, restrictions 1) to 3), it is also necessary to check restrictions 4) to 7). Substituting here the values of the Vickrey discounts:

4) $D_a + D_b = €20 + €10 \geq €10$

5) $D_a + D_c = €20 + €5 = €25$

6) $D_b + D_c = €10 + €5 = €15$

7) $D_a + D_b + D_c = €20 + €10 + €5 < €50$

Accordingly, after applying the Vickrey discounts to the winning bids, restrictions 5), 6) and 7) are satisfied.

The only restriction which is not satisfied is condition 4). Since this requires that the sum of the discounts of A and B are less than or equal to €10, the minimum amount necessary for the satisfaction of restriction 4) must be subtracted from the Vickrey discounts (which are, as mentioned above, the maximum discounts possible). The amount has to be the least possible because, as mentioned above, the objective is to maximize the discounts, subject to the conditions listed. Therefore, as the sum of the Vickrey discounts of A and B is €30, this amount should be reduced by €20.

This requires the application of additional criterion to determine how the €20 will be split between the discounts of A and B. The Regulation of this auction determines that the reduction of discounts with respect to the conditions which are not satisfied (with the application of Vickrey discounts) should be performed by minimizing the squares of the differences between the discounts actually applied to each winning bid and the maximum discount possible (Vickrey discounts), subject as always to the fulfilment of the cited restrictions. This exercise, in the Euclidean space, is equivalent to minimizing the joint distance between the applicable discounts and the Vickrey discounts.

Returning to the example, the only restriction which is not satisfied is restriction 4). As mentioned above, the remaining restrictions continue to be satisfied following the reduction applied to the bids of A and B (i.e. altering the values of the discounts to be applied to the bids of A and B so that restriction 4) is satisfied, does not alter the conclusions with respect to the other conditions). Therefore, the only "active" restriction in this minimization problem is the one which remains unsatisfied. Since the aim is to obtain the highest possible discounts, inequality 4) is transformed into an equation.

As such, we have the following expression to solve:

$$\min_{D_a, D_b} \left[(D_a - DV_a)^2 + (D_b - DV_b)^2\right], \text{ s.a. } D_a + D_b = DV_{a,b}$$

Substituting with the values of the Vickrey discounts and the opportunity cost of A and B:

$$\min_{D_a, D_b} \left[(D_a - €20)^2 + (D_b - €10)^2\right], \text{ s.a. } D_a + D_b = €10$$

Including condition $D_a = €10 - D_b$ in the expression to be minimised, deriving this with reference to the unknown to be determined (D_b) and equalling to zero, the following values are obtained for the two discounts:

D_a = €10;

D_b =€0;

As A bid €35, discounting €10 from A's bid results in the price payable by this bidder (base price - P_a)) € 25. Similarly, since there is no discount for bidder B, bidder B will pay the bid amount, i.e. P_b =€25.

No discount is applied to bid B due to the distance of the amounts of bid A and B in relation to their Vickrey discounts. Because these discount are the same for both bidders, and since the objective is to minimize the distance between each individual discount and the Vickrey discount, an amount needs to be subtracted from the bid of A, since it is that which is strictly further away from the respective Vickrey discount (after the minimization process, , both are at the same distance, and the discount

applied to bidder A can never result in a value payable which is lower than that of bidder B).

On the other hand, there is no place for a reduction in the discount of bidder C, because this reduction would have no impact on the fulfilment of the restrictions - all the restrictions involving C's discount are already met by the Vickrey discount and therefore they remain met subsequent to the alteration of the bids of A and B, since these represent zero or negative variations. Since the aim is to make the discount applied to C as high as possible, while ensuring compliance with the restrictions, this will be the Vickrey discount.

Therefore, the discount applied to C's bid, C, D_c is equal to the Vickrey discount:

D_c = €5

The price payable by C (P_c) is €35.

Example 2

Number of lots: 9

Number of regions: 1

Number of bidders: 5

Bids:

Bidder	Number of lots	Amount bid
A	3	€ 35
B	1	€ 35
C	5	€ 45
D	3	€ 14
E	3	€ 30

As in the previous example, the first step is to determine the winning bids. The set of bids that generates the highest total revenue is the set comprising bids A, B and C, with allocation of all 9 available lots.

The second step is to calculate the base price. To this end, consideration is again given to the restrictions which need to be fulfilled:

1) $D_a \leq DV_a$
2) $D_b \leq DV_b$
3) $D_c \leq DV_c$
4) $D_a + D_b \leq DV_{a,b}$
5) $D_a + D_c \leq DV_{a,c}$
6) $D_b + D_c \leq DV_{b,c}$
7) $D_a + D_b + D_c \leq DV_{a,b,c}$

Using the Vickrey discounts:

1) $D_a \leq €5$
2) $D_b \leq €35$
3) $D_c \leq €15$
4) $D_a + D_b \leq €40$
5) $D_a + D_c \leq €36$
6) $D_b + D_c \leq €36$
7) $D_a + D_b + D_c \leq €71$

It is noted that the Vickrey discount with regard to bidder B is €35, i.e., the total amount bid by this bidder. This occurs because the opportunity cost of bidder B, considered in isolation, is null, i.e. if bidder B did not participate in the auction, and all the other winning bidders remained as they are, none of the non-winning bidders (D and E) could be accommodated, due to that a single lot would become available. However, as seen in the previous example, consideration must be given not only to the opportunity cost of this bidder in isolation, but also the opportunity cost of the sub-sets of bids which include B's bid, so that the discount applied to this bidder does not necessarily have to match the Vickrey discount (which, as seen earlier, is the maximum discount).

Substituting the maximum bids (Vickrey) in restrictions 4) to 7), the following is obtained:

4) $D_a + D_b = €5 + €35 = €40$

5) $D_a + D_c = €5 + €15 < €36$

6) $D_b + D_c = €35 + €15 > €36$

7) $D_a + D_b + D_c = €5 + €35 + €15 < €71$

Restrictions 4), 5) and 7) are satisfied with the Vickrey discounts. Inversely, restriction 6) is not satisfied. Accordingly, it is necessary to minimize the distance between the discounts applied to bidders B and C and the respective Vickrey discounts, subject to which the sum of first amounts to €36.

Replacing the condition in the formula to be minimised, deriving and equalling to zero, results in the following discount values:

D_b = €28

D_c = €8

Meanwhile, the discount applicable to bidder A is the maximum discount possible, given that this reduction would not contribute to the fulfilment of restriction 6). Therefore,

D_a =€ 5

Applying the discounts to the amounts bid, we obtain the following base prices:

P_a = €30

P_b = €7

P_c = €37

The examples alluded above are fairly simple, since only one region is considered, and since each bidder is submitting one bid only. However, it is possible to apply the same reasoning to a more complex example.

Example 3

Number of lots: 4 per region

Number of regions: 4

Number of bidders: 4

Bids:

Bidder A:

Bidding Options	Number of lots				Amount Bid
	Area 1	Area 2	Area 3	Area 4	
A#1	3	2	0	5	€ 2,925
A#2	2	1	0	3	€ 1,400
A#3	3	5	0	4	€ 1,939
A#4	3	1	0	1	€ 1,514

Bidder B:

Bidding Options	Number of lots				Amount Bid
	Area 1	Area 2	Area 3	Area 4	
B#1	3	4	1	0	€ 2,725
B#2	2	3	2	0	€ 1,300
B#3	1	2	3	0	€ 1,439

Bidder C:

Bidding Options	Number of lots				Amount Bid
	Area 1	Area 2	Area 3	Area 4	
C#1	0	1	1	3	€ 1,700
C#2	1	1	1	1	€ 1,100
C#3	0	0	2	1	€ 500

Bidder D:

Bidding Options	Number of lots				Amount Bid
	Area 1	Area 2	Area 3	Area 4	
D#1	2	3	1	1	€ 850
D#2	2	1	2	3	€ 1,150

As in the two previous examples, the first step is to identify the winning bidders. Since, in this example, each bidder makes more than one bid, it is necessary to calculate all the possible bidding combinations, on condition that only one bid is accepted per bidder. Note that the bids A. 1 and A. 3 are rejected because the number of lots available in each geographical area is 4, and these bids were for 5 lots in one of the geographical areas.

The sets of bids eligible to win are:

- (A#4 + B#3 + C#1)=€ 4653;
- (A#2 + B#3 + C#2)=€ 3939;
- (B#1 + C#3)=€ 3225;
- (A#4 + C#1)=€ 3214;
- (A#2 + B#2 + C#3)=€ 3200;
- (B#3 + C#1)=€ 3139;
- (B#2 + C#1)=€ 3000;
- (A#4 + B#3)=€ 2953;
- (A#2 + B#3)=€ 2839;
- (A#2 + B#2)=€ 2700;
- (A#4 + C#2)=€ 2614;
- (C#1 + D#1)=€ 2550;
- (B#3 + C#2)=€ 2539;
- (A#2 + C#2)=€ 2500;
- (B#2 + D#2)=€ 2450;
- (B#2 + C#2)=€ 2400;
- (A#2 + D#1)=€ 2250;
- (C#2 + D#2)=€ 2250;
- (A#4 + C#3)=€ 2014;
- (C#2 + D#1)=€ 1950;
- (A#2 + C#3)=€ 1900;
- (B#2 + C#3)=€ 1800;
- (C#3 + D#2)=€ 1650;
- (C#3 + D#1)=€ 1350;

The set of bids which generates the highest bid amount is (A. 4 + B. 3 + C. 1), totalling €4653. Therefore, this is the set of winning bids. Having determined the winners, the next step is to calculate the base prices.

As in the previous examples, with three winning bidders (A, B and C) three basic restrictions need to be satisfied:

1) $D_a \leq DV_a$
2) $D_b \leq DV_b$
3) $D_c \leq DV_c$

Excluding A's bids, the best possible alternative is the combination (B#1 + C#3), which makes a total of €3225. Since the set of winning bids totals €4653, the maximum discount applicable to A's bid is €1428.

Excluding the bids made by B, the best possible alternative is the combination (A#4 + C#1), whose sum of bids adds €3214. Subtracting this value from the total of the winning bids, the maximum discount is obtained that can be applied to the winning bid of this auction, i.e. €1439.

Excluding the bids made by C, the best possible alternative is the combination (A#4 + B#3), with a total of €2953. The maximum discount applicable to the winning bid of C is €1700.

Substituting these values in restrictions 1), 2) and 3) we obtain the following:

1) $D_a \leq €1428$
2) $D_b \leq €1439$
3) $D_c \leq €1700$

Having identified the Vickrey discounts, we now need to consider the following combinatorial restrictions:

4) $D_a + D_b \leq DV_{a,b}$
5) $D_a + D_c \leq DV_{a,c}$
6) $D_b + D_c \leq DV_{b,c}$
7) $D_a + D_b + D_c \leq DV_{a,b,c}$

Excluding the bids made by A and B, the best alternative is (C#1 + D#1), with a total of €2550. Accordingly, the maximum discount jointly applicable to bids 4 of bidder A and bid 3 of bidder B will be €2103;

Similarly, A and C will together receive a maximum discount of € 2203, and B and C together will receive a maximum discount of € 2403;

Finally, if bidders A, B and C had not entered the auction, D's bids would remain. Of these, the highest value is €1150, and, therefore, the sum of the discounts to be applied to bids of A, B and C cannot exceed €3503. Accordingly, the following conditions are obtained:

4) $D_a + D_b \leq €2103$
5) $D_a + D_c \leq €2203$
6) $D_b + D_c \leq €2403$
7) $D_a + D_b + D_c \leq €3503$

Replacing the Vickrey discounts in restrictions 4) to 7) yields:

4) $D_a + D_b = €1428 + €1439 > €2103$

5) $D_a + D_c = €1428 + €1700 > €2203$

6) $D_b + D_c = €1439 + €1700 > €2403$

7) $D_a + D_b + D_c = €1428 + €1439 + €1700 > €3503$

In this example, none of the four combinatorial restrictions are satisfied with the Vickrey discounts. As seen in the previous examples, the next step is to minimize the distance in the Euclidean space between the applicable discounts and the Vickrey discounts, subject to the restrictions which are not satisfied. However, in this particular situation, it is not necessary to proceed with this exercise. As seen above, if any condition is not satisfied, the way to ensure that discounts are as high as possible is to turn inequality into equality - if the conditions were satisfied below the maximum value, the discounts would not be maximised. Transforming the inequalities 4) to 6) into equations, 3 equations and 3 unknowns remain, and therefore the system can be solved. If the vector of identified discounts satisfies restriction 7), then this vector is the one that maximizes the discounts, subject to the satisfaction of the specified restrictions. In this situation, there is no need to minimize the distance of the applicable Vickrey discounts, because the solution found is unique, and as such, is also minimal. This results in the following system:

$$\begin{cases} D_a + D_b = €2103 \\ D_a + D_c = €2203 \\ D_b + D_c = €2403 \end{cases}$$

The solution to the system is the following:

D_a = €951.5

D_b = €1151.5

D_c = €1251.5

Replacing in condition 7), the following is obtained:

$$D_a + D_b + D_c = €951.5 + €1151.5 + €1251.5 < €3503$$

As the restriction is satisfied, the vector (€ 951.5, € 1151.5; € 1251.5) is the one that maximizes the discounts applicable to each winning bid, subject to the restrictions which ensure that the winning bidders, when considered individually and in subsets, remain as winning bidders.

The base prices to be paid are as follows:

P_a = €562.5

P_b = €287.5

P_c = €448.5

5.3 Resolution of draws in the distribution stage

In the event of a tie in the bidding round of the distribution stage, this will be resolved through the successive application of the following criteria:

1. The group of bids that results in the greatest number of regions to be allocated shall be selected; in the event of a continuing tie:
2. The group of bids that results in the greatest number of winning bidders shall be selected; in the event of a continuing tie:
3. The group of bids that results in the largest number of lots to be allocated shall be selected; and finally, in the event of a continuing tie:
4. This will be resolved through a random draw among the tied groups of bids, to be held at a place and date to be decided by the Committee (established under article 5 of the Regulation).

6 Assignment Stage

The assignment stage involves, if necessary, a bidding round, a process of lot selection and, where relevant, a time period in which bidders can exchange their blocks.

The objective of the assignment stage is to determine how the available lots in each one of the two sub-bands of the 3400-3600 MHz frequencies (lots A and B) and 3600-3800 MHz (lots C and D) will be distributed among the winning bidders determined in the distribution stage, as well as the final prices to be paid.

The bidding round takes place whenever, in a given geographical area, there is a possibility of "dispute" in the choice of lots, or in other words, when there are at least three bidders, or two bidders where one bidder has won at least two lots (in a given geographical area).

The lot selection only takes place in the following situations:

1) In the case of a single bidder winning up to 3 lots after the bidding round in the distribution stage;
2) If only two bidders win individual lots after the bidding round in the distribution stage;
3) Where a single bidder wins a single lot in one of the frequency sub-bands, and there are one or two winning bidders in the other frequency sub-band after the bidding round in the assignment stage;
4) In the case of two bidders winning individual lots in the same frequency sub-band after the bidding round in the assignment stage.

It should be noted that during the distribution stage the number of lots to be assigned to each winning bidder in each geographical area will have already been determined, as well as the base price to be paid; however, the specific frequency sub-bands will not have been determined.

6.1 Bidding round in the assignment stage

Bidders may submit bids for every possible option using the personalized form provided by the Committee;

The number and type of bidding options available on each bidder´s form depends on the number of geographical areas where the bidder has been determined as one of the winners;

Articulating article 25 with articles 23 and 24 of the Regulation, bidders will submit a bidding form to the Committee, in which the number of bidding options depends on the number geographical areas where the bidder has won lots and the number of lots won by other bidders in those areas.

It should be stressed that, for this bidding round, the bidder is under no obligation to bid. For example, having no sub-band preference, with regards to the assignment of the number of lots won in the previous stage, the bidder can choose not to submit the bidding form during the assignment stage.

The bidder is also not required to submit a bid for all available options, if he so chooses. According to paragraph 4 of article 24, in the event of not all the bidding options being submitted, the respective top-up value for this bid shall be considered as zero.

In this round bidders may, in each bid:

- Express their preference for frequency sub-bands 3400-3600 MHz or 3600-3800 MHz, as follows:
  - The bidder must mark the relevant box for each region, entering the value '1', stating that they prefer the lot(s) in the frequency sub-band 3400-3600 MHz, or
  - The bidder must leave the box blank or entering the value '0', stating that they prefer the lot(s) in the 3600-3800 MHz frequency sub-band.
- Indicate the bid top-up value expressed in thousands of euros for each of the preferences expressed.

Expressing their preference, the bidder has the following options:

- If the bidder wins a single lot in a given geographic area, a preference may be revealed between lots of the 3400-3600 MHz sub-band (A or B) and the lots of the 3600-3800 MHz sub-band (lots C or D);

- If the bidder wins two lots in a given geographic area, a preference may be revealed between lots of the 3400-3600 MHz sub-band (A or B) and lots of the 3600-3800 MHz sub-band (lots C or D);

- If the bidder wins three lots in a given geographic area, a preference may be revealed between the set of lots A, B and C and the set of lots B, C and D.

Bidders may submit multiple top-up bid values for the different combinations of their preferences in the relevant geographical areas;

The following table shows an example of a top-up bid, for the case where a particular winning bidder:

- Won lots in geographical areas 1, 2, 4 and 9;
- In geographical area 1, the bidder, as the sole winner, will not make any top-up bid;
- On the other hand the bidder has a slight preference for acquiring lots in the 3400-3600 MHz frequency band (revealed by marking the box with a '1' in the first 4 bids) and their preference is strongest in region 2 (according to the submitted value of the top-up bid).

Assignment Bidding Form										
Region	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	
Number of lots won	3	2	0	3	0	0	0	0	1	
Number of lots won by other participants	0	2	3	1	2	1	3	4	2	Amount of top-up bid (thousands of euros)
Bid No		Preference for A+B		Preference for A+B+C					Preference for A/B	
1	N/A	**1**	N/A	**1**	N/A	N/A	N/A	N/A	**1**	1 000
2	N/A	**1**	N/A	**1**	N/A	N/A	N/A	N/A	**0**	900
3	N/A	**1**	N/A	**0**	N/A	N/A	N/A	N/A	**1**	800
4	N/A	**1**	N/A	**0**	N/A	N/A	N/A	N/A	**0**	500
5	N/A	**0**	N/A	**1**	N/A	N/A	N/A	N/A	**1**	600
6	N/A	**0**	N/A	**1**	N/A	N/A	N/A	N/A	**0**	200
7	N/A	**0**	N/A	**0**	N/A	N/A	N/A	N/A	**1**	200
8	N/A	**0**	N/A	**0**	N/A	N/A	N/A	N/A	**0**	0

6.2 Determination of the winning bids during the assignment stage

Through the process of determining the winning bids, only one bid is selected from each bidder among those listed on the form (including those which, having been omitted, are automatically assigned with a top-up bid amount of zero).

When the bids of all bidders are accommodated in terms of preferences by the sub-bands in each geographical area, bid combinations will be obtained as probable winners. The winning combination is chosen according to the criterion of the highest generated value among all others, in accordance with articles 24 and 25.

An example is presented below which reflects the top-up bids that bidders X, Y and Z have submitted in the bidding round of the assignment stage.

Bidder X:

Bidding Options	Preferences				Top-up bids €
	Area 1	Area 2	Area 3	Area 4	
No of lots won X	3	1	0	1	
No of lots won by others	1	3	4	3	
Preference	A+B+B	A/B		A/B	
X#1	1	1	N/A	1	100
X#2	1	0	N/A	0	50
X#3	0	1	N/A	1	50
X#4	0	0	N/A	0	100

The bids presented in the above table indicate that Bidder X has a strong preference for the assignment of spectrum in the same sub-band in different areas (X#1 e X#4). Furthermore, in the event that Bidder X does not succeed in his first preference, the lower value bids indicate that he places little value on the assignment of spectrum in the specific band in area 1, provided that he succeeds in acquiring spectrum in the same sub-band in areas 2 and 4 (X#2 and X#3).

Bidder Y:

Bidding Options	Preferences				Top-up bids €
	Area 1	Area 2	Area 3	Area 4	
No of lots won Y	1	2	3	0	
No of lots won by others	3	2	1	4	
Preference	A/B	A+B	A+B+V	N/A	
Y#1	1	1	1	N/A	90
Y#2	1	1	0	N/A	60

The bids presented in the above table indicate that Bidder Y prefers an assignment of spectrum in the lower sub-band for the areas where he obtains spectrum, a preference which is slightly greater in areas 1 and 2.

Bidder Z:

Bidding Options	Preferences				Top-up bids €
	Area 1	Area 2	Area 3	Area 4	
No of lots won Y	0	1	1	3	
No of lots won by others	4	3	3	1	
Preference	N/A	A/B	A/B	A+B+C	
Z#1	N/A	0	0	0	80
C#2	N/A	1	1	1	85

The bids presented in the above table indicate that Bidder Z prefers an assignment of spectrum in the same sub-band, with a slight preference for the lower sub-band.

In this example, since all bidders have won lots in at least 3 areas, each bidder should have provided 8 bidding options, in order to cover all the combinations. Where these are not presented, they shall be considered to exist (i.e., they will be created), but with zero value.

Accordingly, in total, there are 512 combinations of bidding, which briefly:

- (X#4 + Y#1) = € 190;
- (X#4 + Y#2) = € 160;
- (Y#1 + Z#1) = € 170;
- (X#1) = € 100;
- (Z#2) = € 85;
- (X#2) = € 50;
- (X#3) = € 50;
- (Y#2) = € 60;
- (Z#1) = € 80;

As a result, the bidders will receive the following combinations of lots:

- X:

Areas	Area 1	Area 2	Area 3	Area 4
No of Lots	3	1	0	1
Lots	B+C+D	C or D	N/A	D

- Y:

Areas	Area 1	Area 2	Area 3	Area 4
No of Lots	1	2	3	0
Lots	A	A+B	A+B+C	N/A

- Z:

Areas	Area 1	Area 2	Area 3	Area 4
No of Lots	0	1	1	3
Lots	N/A	C or D	D	A+B+C

As described in the above section on determining the winners in the distribution stage, in order to calculate the top-up prices of the bidding round of the assignment stage, the first step is to identify the relevant restrictions. The big difference is that, as all bidders in this stage are already winners of a certain number of lots, when the alternative combinations are considered, in order to calculate the opportunity costs, it is always necessary to consider one bid for each bidder - it is also of note that the calculation of the opportunity cost of a given bid will have to include another bid from that same bidder.

For example, if the winning bid of bidder X is no longer so, the best alternative would be the combination of bids Y1 and Z1. Since no bid of X alternative to bid 4 can be accommodated with the bids of Y and Z, another bid which can be accommodated will be chosen. This bid, as it is not entered on the bidding form, will be assigned with a zero amount. Extending the reasoning to all other restrictions, the following table is obtained:

Excluded bids	Alternative bids	Value payable €	Maximum discount €
X	Y1 + Z1	170	20
Y	X4 + Zi^{1} X1 + Zi	100 + 0= 100	90
Z	X4 + Y1	190	0
X+Y	Z2	85	105
Y+Z	X1 ou X4	100	90
X+Z	Y1	90	100
X+Y+Z	X1 + Yi + Zi^{2}	100	90

Note 1: the alternative options to Y are the options X1 or X4, combined with the bids which Z did not enter (zero value).

Nota 2: the alternative options to (X4 + Y1 + *Zi*) is the bidding option of bidding X1 combined with the bids not presented by bidders Y and Z (and considered as being zero).

Applying the maximum individual discounts to the combinatorial restrictions, it is concluded that condition 4 is not satisfied. As such, it is necessary to minimize the distance between the applicable discounts and the maximum individual discounts, subject to the satisfaction of this condition. The minimization produces the following results:

Result €	Top-up prices €
Discount X = 17.5	82.5
Discount Y = 87.5	2.5
Discount Z = 0	0

- Given the results, the lot selection in Area 2 is required (involving bidders X and Z); meanwhile Y already knows the lots it will receive:

  - Y:

Areas	Area 1	Area 2	Area 3	Area 4
No of Lots	1	2	3	0
Lots	A	A+B	A+B+C	N/A

- According to paragraph 2 of article 27, and taking into account the map of geographical areas in Annex 1, a list of bidders will be drawn up, ranked by the largest number of adjacent regions where each bidder has won lots, which in this example will be:

  - X = 2 adjacent areas (2 and 4);

6.3 Resolution of ties in the assignment stage

A tie between winning bids in the bidding round shall be considered as existing when there are several combinations of bids which generate the same amount. In such cases, the procedure set out in paragraph 2 of article 25 for settling a tie shall be applied, which is by random draw.

6.4 Final price associated with lots won

The final price that each bidder will pay for the lots won shall be the sum of the amounts calculated in the two bidding stages, i.e., in the bidding of the distribution stage- base price - and in the bidding round of the assignment phase – top-up price.

The base price may in no case be less than the amount equal to the sum of reserve prices of lots won, while the additional price may be zero.

The base prices, one for each bidder are determined simultaneously by applying the principle of the second price rule, which calculates the bid discount to be subtracted from the winning bids, subject to the following conditions:

- The determination of the winner produces the same result;
- The bid discount for each bidder is not greater than the difference between the value of the winning bid and the sum of the reserve prices associated with the lots of the winning bid.

The top-up prices, one for each bidder participating in the bidding of the assignment stage, are calculated using the second price rule.

The Committee shall notify each bidder of the final price to be paid, including a breakdown of the corresponding base price and additional price, as defined in article 26.

6.5 Lot selection

In the event that a lot selection process is needed, pursuant to paragraph 1 of article 27, a list of winning bidders shall be drawn up. The list will be ranked by the largest number of adjacent regions where each bidder has won lots.

Should two or more bidders have won lots in the same number of geographically adjacent regions, a random draw shall determine their position in the ranking. The draw will take place at a location and date to be decided.

The geographically adjacent area, are a maximum of seven and are those that share at least one border with another region where the same bidder has won other lots.

Imagine that there are three bidders: X, Y and Z.
Bidder X has won lots in Areas 1, 2, 3, 4 and 7, of which four areas are geographically adjacent (areas 1 to 4).
Bidder Y has won lots in Areas 4, 5, 6, 8 and 9, of which three areas are geographically adjacent (areas 4 to 6).
Bidder Z has won lots in Areas 2, 5, 6, 8and 9, of which two areas are geographically adjacent (areas 5 and 6).
Bidder X shall be ranked in first place as a result of winning lots in the largest number of geographically adjacent areas. Bidder Y shall be ranked in second place as a result of winning lots in the second largest number of geographically adjacent areas; and bidder Z hall be ranked in third place as a result of winning lots in the fewest geographically adjacent areas

The Committee shall notify the winning bidders of the procedure to be followed and the deadline established for the choice of the lots to be assigned according to the established list.

Following the ranking in the list drawn up by the Committee, bidders must choose their preferred lots in each geographical area, wherever the selection of lots has not already been determined automatically in the distribution stage and/or in the bidding round of the assignment stage.

If the bidder is the sole winner in a determined geographical area, his selection options are as follows:

- If he wins three lots, he may choose between lots A, B and C and lots B, C and D;

- If he wins two lots, he may choose between lots A and B and lots C and D;
- If he wins a single lot, he may choose between lot A, B and C and D;

As shown in the example given in the "Determination of the winning bids during the assignment stage", when there is more than one bidder in a geographical area, the options for each bidder depend on the outcome of the assignment bidding round, while in some cases the choice of lots is determined automatically, such as, for example, the lots won by bidder X in area 4 which has to be "Lot D" and not "Lot C" since bidder Z won 3 lots in the same area, A+B+C.

6.6 Spectrum swap

Once the lot selection is concluded, the Committee will notify all winning bidders of the number of lots and respective frequency bands assigned in each geographical area.

The Regulation sets out that bidders may swap assigned blocks or lots, according to the following conditions:

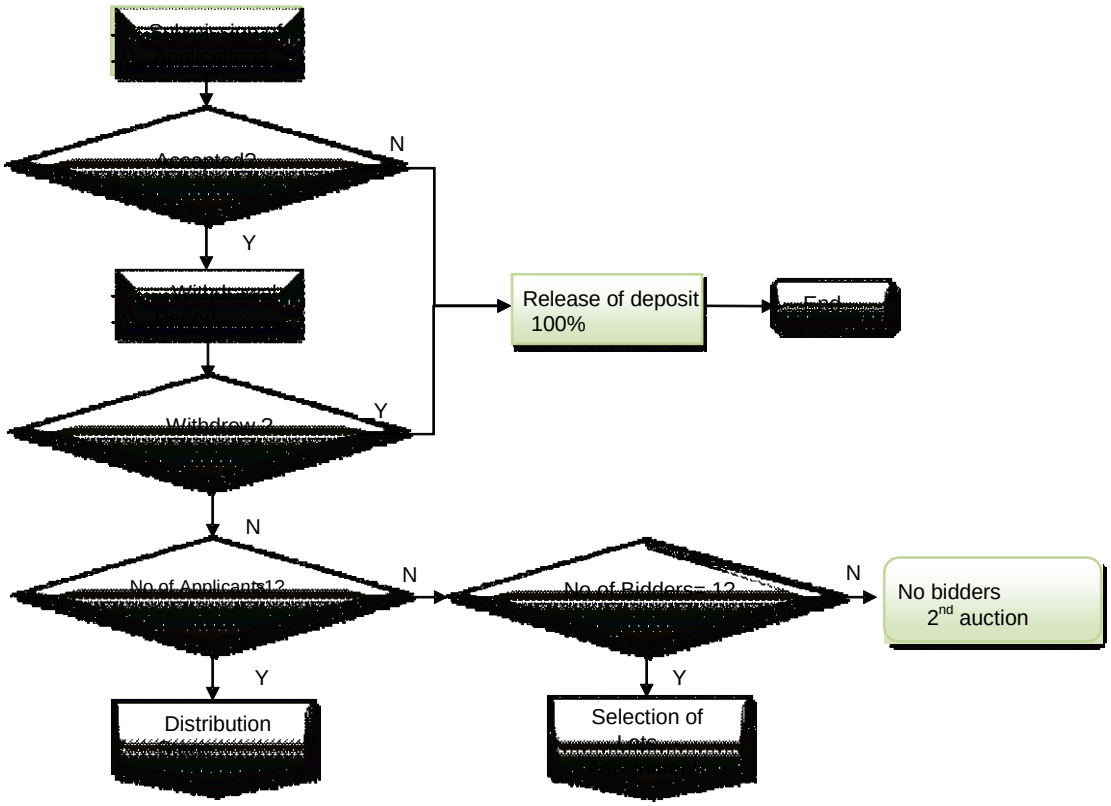
a. None of the winning bidders may obtain a quantity of spectrum different from that which was assigned to it in each one of the given geographical areas;

b. Blocks or lots pertaining to different geographical areas may not be swapped;

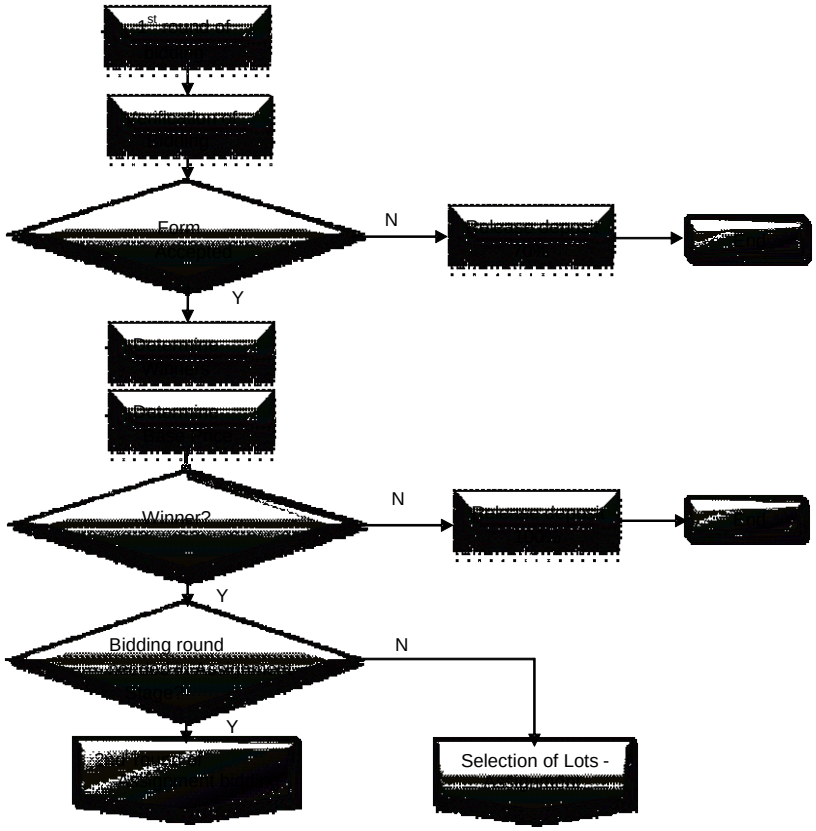
Accordingly, the parties shall notify the Committee within 5 days of their intention, presenting for this effect a declaration signed by the parties involved. It should be noted that the indicated time period is counted from the date on which the results are disclosed in accordance with paragraph 4 of article 26 or paragraph 9 of article 27, where applicable.

7 Auction Flowchart:

7.1 Qualification stage



7.2 Distribution Phase



7.3 Assignment Stage